BIBLIOGRAPHY

1. Gogosov, V. V., Polianskii, V. A., Semenova, I. P. and Iakubenko, A. E., Equations of electrohydrodynamics and transfer coefficients in a strong electric field. Izv. Akad. Nauk SSSR, Mekh. Zhidk. i Gaza, №2, 1969.
2. Germain, P., Mechanics of Continuous Media (Russian translation), Moscow, "Mir", 1965.
3. Ushakov, V. V., On the construction of approximate solutions of two-dimensional potential problems in electrohydrodynamics. Izv. Akad. Nauk SSSR, Mech. Zhidk. i Gaza, №4, 1972.
4. Grabovskii, V. I., Some questions of investigating electrohydrodynamic jets behind the exit of a charged particle source. Izv. Akad. Nauk SSSR, Mekh. Zhidk. i. Gaza, №1, 1972.
5. Buchin, V. A., Problem of an electrohydrodynamic probe which does not disturb the distribution of current density and space charge. PMM Vol. 36, №3, 1972.
6. Vatazhin, A. B., Likhter, V. A. and Shul'gin, V. I., Investigation of an electrogasdynamic jet behind a charged particle source. Izv. Akad. Nauk SSSR, Mekh. Zhidk. i Gaza, №5, 1971.
7. Bortnikov, Iu. S. and Rubashev, I. B., Some questions of investigating the system of equations of electrogasdynamics. Magnitn. Gidrodinam., №2, 1968.
8. Zykov, V. A., Elements of the electrodynamics of unipolar gas flows. Teplofiz. Vysok. Temper., Vol. 7, №6, 1969.
9. Grabovskii, V. I., Electrohydrodynamic flow in a plane channel with axial electrode-emitter. Prikl. Mekh. i Tekh. Fiz., №6, 1970.
10. Lavrent'ev, M. A. and Shabat, B. V., Methods of the Theory of Complex Variable Functions, 3rd ed. Moscow, "Nauka", 1965.

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UDC 538.4

THE SPREADING OF SINGLY IONIZED JETS IN HYDRODYNAMIC STREAMS

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This investigation pertains to the spreading of singly ionized [unipolarly charged] jets in hydrodynamic streams applicable to problems of electrohydrodynamic flows downstream of a source of charged particles ("free" jets), in the channels and ducts of electrohydrodynamic systems ("enclosed" jets). Basic nondimensional parameters have been defined, upon which the intensity of spreading of the jets depends. By means of a numerical solution of the two-dimensional equations of electrohydrodynamics the distribution of the electric parameters (charge density, electric potential) in the jet and in the surrounding space has been established.

Electrohydrodynamic flows in space have been studied before in quasi-one-dimensional approximation [1] in linear approximation [2, 3], under the assumption that the mobility of the charged particles be zero [4] and for arbitrary mobility of the particles in the case of fluid motion in a channel [5]. An experimental investigation of the speading of singly ionized jets downstream of a source of charged particles was carried out in [6]. In the papers enumerated (aside from 1 and 6) flows were studied in which the region of the hydrodynamic flow coincided with the region occupied by the ionized component. However in practice conditions are encountered in which the electrically charged fluid occupies a region smaller than the entire extent of the flow. This results from the fact that the charged particles enter the stream from emitter-electrodes which ordinarily occupy only a portion of the entrance section of the channel. From this the investigation of "electric jets" derive their importance and by this designation will be understood; from here on, the regions within the hydrodynamic stream which carry an an electric space charge different from zero.

In the present paper the basic characteristics of the spreading of electric jets are clarified under various conditions. Therefore, in the analysis of such problems it is essential to take into account the nonhomogeneity of the electric field in the region of the charged jet as well as in the hydrodynamic stream not carrying charges and in the surrounding space.

1. In many applications (jets downstream of a source of charged particles, motion of a charged fluid in duct systems, flows in certain regions of electrohydrodynamic generators and propulsion plants) where electrohydrodynamic motions are described, the variation of hydrodynamic quantities due to the action of electric forces, can be disregarded. In this case the problem is reduced to the determination of electrodynamic parameters, i.e. of the electric space charge density q, the electric field $\mathbf{E}$, the electric potential φ and the electric current density $\mathbf{j}$ in a given field of hydrodynamic velocity $\mathbf{V}$. With the assumption of steady flow of the charged particles b the pertinent system of equations has the form [6]

$$\mathbf{j} = q(\mathbf{V} + b\mathbf{E}), \qquad \operatorname{div}\mathbf{j} = 0, \quad \operatorname{div}\mathbf{E} = 4\pi q\varepsilon^{-1}, \quad \mathbf{E} = -\operatorname{grad}\varphi \quad (1.1)$$

Here $\varepsilon = \text{const}$ is the dielectric constant. The system (1.1) is equivalent to the following two equations:

$$\Delta\varphi = -4\pi q\varepsilon^{-1} \quad (1.2)$$

$$(\mathbf{V} - b\nabla\varphi)\nabla q + (\operatorname{div}\mathbf{V} + 4\pi bq\varepsilon^{-1})\,q = 0 \quad (1.3)$$

Equation (1.2) is an elliptical equation, and Eq. (1.3) is hyperbolic in q (first order) the characteristics of which represent the trajectories of motion of the charged particles. These particularities of the equations determine the establishment of contacting boundary conditions in various specific problems.

In what follows two classes of problems are investigated representing models of the flow of charged fluid in open space (free jets) and in channels (enclosed jets).

2. Let us consider the spreading of a plane electric jet (occupying the region G) within the hydrodynamic jet penetrating between infinite electrode screens $x = 0$, $x = L$ (where x = longitudinal coordinate) which are "pervious" for the fluid and do not disturb the hydrodynamic flow.

The dimensions H and h of the hydrodynamic and the electric jet in the initial section ($x = 0$) satisfy the inequality $h < H$. In this manner the region to be investigated is represented by the slice $0 \leqslant x \leqslant L$, $|y| < \infty$.

Let the potentials of the electrodes $x = 0$ and $x = L$ be zero. Then it is necessary at $|y| \to \infty$ to make use of the asymptotic condition $\varphi \to 0$. Note that in practical calculations the asymptotic condition must be applied on lines $|y| = A \gg 1$ located sufficiently far away from the axis of the jet. The extent of the electric jet in section $x = 0$ is given by establishing the boundary condition

$$q = q_* = \text{const} \quad \text{for } |y| \leqslant h/2, \quad q = 0 \quad \text{for } |y| > h/2 \qquad (1.4)$$

On the boundary Γ of the electric jet (of the region G) there occurs a discontinuity of the charge q, this charge being zero outside the jet and different from zero inside G. This corresponds to a discontinuity of the second derivative of the potential. Therefore it is necessary to obtain the solutions of the equations within and outside the region G and to match the solutions on the boundary Γ of the electric jet. Since in the problem as formulated here there does not exist a surface charge on the boundary Γ, the condition of continuity of the potential and of the electric field serve as matching conditions.

Let us change over to nondimensional quantities by the formulas

$$\varphi = v_* H b^{-1} \varphi^\circ, \quad q = \varepsilon v_* (4\pi b H)^{-1} q^\circ, \quad \mathrm{V} = v_* \mathrm{V}^\circ, \quad x = H x^\circ, \quad y = H y^\circ \quad (1.5)$$

In nondimensional variables the system of equations (1.2), (1.3) with the assumption $\operatorname{div} \mathrm{V} = 0$ is changed into the form (omitting further the superscript zero for the nondimensional quantities)

$$\Delta\varphi = -q \qquad (1.6)$$

$$\frac{\partial q}{\partial x}\left(u - \frac{\partial \varphi}{\partial x}\right) - \frac{\partial q}{\partial y}\frac{\partial \varphi}{\partial y} + q^2 = 0 \qquad (1.7)$$

From the statement (1.7) it is seen that in the flows under consideration the cross-stream velocity is much smaller than the axial velocity u. The distribution of the velocity u was chosen of the form

$$u = \sqrt{\frac{3}{3+x}}\left[1 - \left(\frac{2y}{1+0.3x}\right)^{3/2}\right]^2 \qquad (1.8)$$

This type of function defines the decrease of speed in the x direction and its approach to zero at the edge of the hydrodynamic jet

$$y_e = (1 + 0.3\,x)/2$$

The system of boundary conditions in nondimensional variables has the form

$$\begin{aligned} &\varphi = 0 \quad \text{for } x = 0, \quad x = l \quad (l = L/H) \\ &\varphi = 0 \quad \text{for } y = \pm A^\circ \quad (A^\circ = A/H) \\ &q = \beta \quad \text{for } x = 0, \quad |y| \leqslant a/2 \\ &(a = h/H, \quad \beta = q_* 4\pi b H (\varepsilon v_*)^{-1}) \end{aligned} \qquad (1.9)$$

The problem is governed by three parameters, namely by the geometric factors l, a and by the quantity β. (The parameter A° was chosen sufficiently large so that it has no influence upon the distribution of the parameters in the region investigated). From now on we take $l = 1$ and shall investigate the widening of the electric jet for

$a < 1$. Since the stream is symmetrical with respect to the axis $y = 0$, we need to study the problem only for the region $y \geqslant 0$, making use of the conditions of symmetry $\partial\varphi/\partial y = 0$ for $y = 0$.

The solution of the problem formulated in this manner was carried out numerically by the method of successive approximations described in [5]. This method comprises the successive solution of the elliptic and the hyperbolic equations (1.6) and (1.7). In this process Eq. (1.7) was analyzed utilyzing the method of characteristics. The characteristics of that equation are the lines

$$\frac{dy}{dx} = -\frac{\partial\varphi / \partial y}{u - \partial\varphi / \partial x} \tag{1.10}$$

along which the following relation is valid:

$$dq = -q^2 \frac{dx}{u - \partial\varphi / \partial x} \tag{1.11}$$

For each step, first of all the boundary Γ of the region G within which $q \neq 0$ was determined by the method of straight characteristics. Then, within this defined region the value of q was calculated by the method of inverse characteristics for each grid point of the rectangular computational net utilized in the solution of the elliptic equation in the preceding stage. The process of iteration was carried through until the difference between two successive approximations was less than the fourth power of the step of the computational net.

The results of the computations will now be given. The upper half of the region of flow $0 \leqslant x \leqslant 1$, $0 \leqslant y < \infty$ is shown in Fig. 1. Line *1* represents the boundary of the hydrodynamic stream, lines *2* and *2'* the extent Γ of the electrical jets for $a = 0.2$ and $a = 0.4$, respectively. The lateral expansion of the electrical jet grows with increase of the parameter β. Physically this is related to the fact that three factors, namely the decrease of hydrodynamic velocity, the increase of the charge in the region of flow, and the intensification of mobility of the charged particles bring about a more intensive spreading of the jet. For $\beta = 0$, i.e. when the mobility $b = 0$, the jet does not spread at all. When β increases in the range $\beta > 10$ the boundary of the jet is inappreciably deformed. This is an indication of the existence of a mode of "collecting" (*) in which case all characteristics, among them also the boundary Γ of the electric jet, cease to depend upon the magnitude of the charge q_* on the emitter. The spreading velocity of the electric jet grows also with the increase of the parameter a. Thus, if $\beta = 2$, for $a = 0.2$ the jet gains 80 percent in thickness and

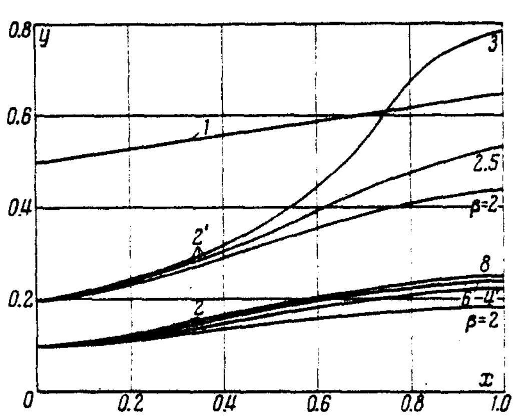

Fig. 1

*) Editor's Note. Acquisition of fluid by induction.

for $a = 0.4$, 120 percent. This is explained by the fact that the peripheral parts of the wider jets extend to regions with smaller velocity u and, further, that with an increase of the quantity a in the field of flow there arise cross-stream electric fields on account of the larger charge imposed by the emitter. Let us note that there exist modes for which the electric jet is larger than the hydrodynamic, and in these regions the motion of the charged particles proceeds only under the action of the electric field. The spreading of the electric potential in the cross-stream section $x_0 = 0.4$, for $a = 0.2$ (solid line) and $a = 0.4$ (dashed line) for various values of β is shown in Fig. 2. The maximum values of the potential occur on the axis of the jet. The flow is particular for the reason that the electric potential differs from zero in all three regions, namely in the region of the electric jet G, in the parts of the hydrodynamic stream outside G and in the outer space. With increase of a and β the potential of the axis increases.

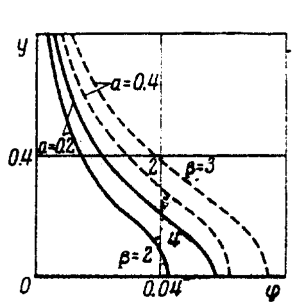

Fig. 2

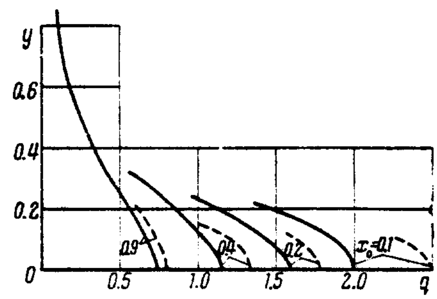

Fig. 3

The diagrams of electric charge density in various sections ($x_0 = \text{const}$) for $a = 0.2$, $\beta = 4$ (dashed lines) and $a = 0.4$, $\beta = 3$ (solid lines), are presented in Fig. 3. It is noteworthy that the maximum of q, occurring on the axis of the jet, decreases with increasing x_0. With increase of β the maximum charge increases. The opposite trend is observed for an increase of the parameter a, because a growing axial electric field inhibits the emission of charges. Along each line $y = \text{const}$ lying within the region of the jet G the charge density decreases monotonously with longitudinal distance along the jet and the distribution of the potential shows up a maximum which is displaced slightly from the middle of the interelectrode distance toward the emitter-electrode.

3. Let us investigate the particularities of spreading of the enclosed electric jet. For this purpose we consider the plane electrohydrodynamic flow in a channel $0 \leqslant x < \infty$, $|y| \leqslant 0.5$ with ideally conducting walls held under an identical potential $\varphi = 0$. (Here the height of the channel serves as the characteristic dimension). Let there be located at the inlet section $x = 0$ an electrode screen pervious to the fluid which is electrically connected with the walls and which has the potential $\varphi = 0$. In practical application the area of the emitter of charged particles ordinarily is less than the area of the inlet section of the channel. In order to take account of this fact we shall assume that the emitter occupies the portion $x = 0$, $|y| \leqslant a/2 < 0.5$, throughout which charged particles are imposed upon the stream at constant density. The velocity of the fluid is assumed constant throughout the channel, $u = 1$. The spreading of an electric jet of this kind is conditioned by the "initial" subregion, in which the

jet has not yet reached the walls of the channel and has "free" edges. In this subregion is contained the total streamwise electric current $J = J_0$, where J_0 is the electric current at $x = 0$ (the "imposed current" from the emitter). The jet reaches the walls of the channel after some distance sa from the entrance section $x = 0$ and we have $q \neq 0$ in all sections of the channel for $x \geqslant sa$ (in the "basic" subregion). (Here s is the ratio of the length dimension of the initial subregion with respect to the width of the electric jet h in the entrance section). The electric current J, for $x > sa$ decreases since the current density $j_y = -q\, \partial\varphi / \partial y$ differs from zero on the wall. The most intensive decrease of J takes place at the start of the basic subregion. For $x \gg sa$, when the cross-stream electric fields become inappreciable on account of the low values of q, the value of J changes inappreciably.

Mathematically the problem then reduces to the integration of the system of equations (1.6), (1.7) with the boundary conditions

$$\varphi = 0 \quad \text{for } y = \pm 0.5, \qquad 0 \leqslant x < \infty$$
$$\varphi = 0 \quad \text{for } x = A^\circ, \quad |y| \leqslant 0.5 \quad \text{for } x = 0,\ |y| \leqslant 0.5$$
$$q = \beta \quad \text{for } x = 0, \quad |y| \leqslant a/2 \quad (\beta = q_* 4\pi b H (e v_*)^{-1}) \tag{3.1}$$

The numerical solution of the system (1.6), (1.7), (3.1) was of the same type as in Sect. 2. The section $x = A^\circ$ was placed sufficiently far downstream and has no influence upon the distribution of the electric parameters in the initial and the basic subregions. The solution of the problem depends upon two dimensionless parameters a and β which vary over a sufficiently large range.

The particularities of the development of the electric jet are illustrated graphically in Fig. 4, where the upper boundary of the jet $\Gamma(x)$ for various a and β, is shown.

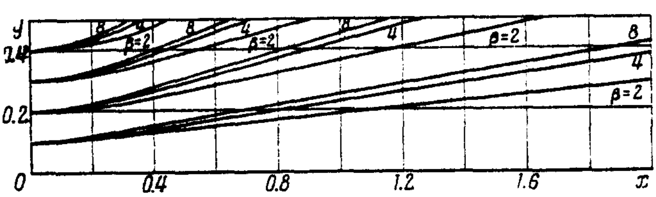

Fig. 4

The intensity of lateral growth of the jet increases with a and β. For $\beta = 0$ (a flow of this type, as also in Sect. 2 can be regarded as the motion of charged particles with zero mobility) the electric jet does not spread at all. For sufficiently large β ($\beta > 10$) the shape of the jet changes inappreciably, which is evidence that the regime of inductive acquisition of fluid prevails. For large a the electric jet grows in width more intensively. The dependence of the length of the initial subregion of the jet $s = s(a, \beta)$ is shown in Fig. 5. With the growth of the parameter a and β there is a sizeable decrease of the value of s. On account of this the dependence of the parameter β upon the dimension of the initial subregion becomes weaker with increase of a. This is related to the fact that for $a \approx 1$ the charged particles reah the wall of the channel very fast, for one thing because of their closeness to these walls and for another because of the higher cross-stream potential gradient which arises for sufficiently large a-values.

In Fig. 6 are presented the distribution of the potential (solid lines) and of the electric charge density (dashed lines) along the axis of the channel for various β and $a = 0.6$. The electric potential and the charge rise with increase of β. The charge density diminishes monotonously along the axis from its maximum value for $x = 0$, and the potential initially rises, attains a maximum and thereafter decreases asymptotically to zero. It is interesting that for all β the maximum in the axial distribution in the potential occurs near the section $x = 0.5$. The same is true also for all other a. An increase of a leads to a rise of the potential and in consequence thereof, to a decrease of the charge. Electric charge density diagrams of q in various sections (x_0 = const) of electric jets for

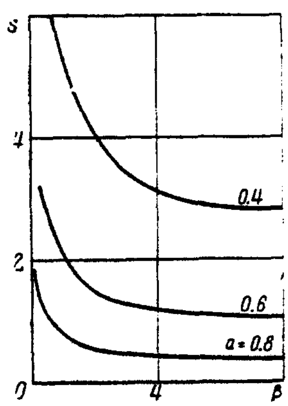

Fig. 5

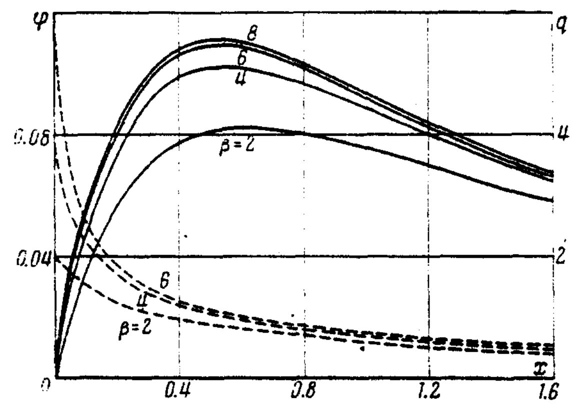

Fig. 6

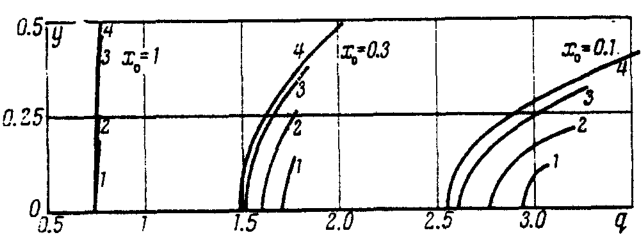

Fig. 7

$\beta = 8$ and different a are shown in Fig. 7. (Curves 1—4 are plotted for $a = 0.2, 0.4, 0.6, 0.8$, respectively). The value of q rises monotonously from the axis of the jet in direction toward its edge (in Fig. 7 only one-half of the channel $y \geqslant 0$) is shown). The occurrence of a minimum on the axis of the jet is a typical particularity of enclosed electric jets. (Let it be recalled that in open jets, (Sect. 2), the maximums occurred on the axis). With increasing distance from the initial section the nonhomogeneity in the distribution of q decreases. For $x_0 = 1$ the space charge practically does not change across the electric jet. Let us note that the value of q in this and the subsequent sections does not depend upon a although the thickness of the electric jets in that case is different.

The curves in Fig. 8 show the imposed current J_0 as a function of the parameters a and β. With the growth of these quantities the current increases monotonously. An interesting particularity of the flow is revealed in case of inductive acquisition of fluid, in which case J_0 ceases to depend upon β if $\beta > 10$. The variation of the current $J(x)$, conducted by the electric jet along the channel is

$$J(x) = \int_{-0.5}^{0.5} q\left(1 - \frac{\partial \varphi}{\partial x}\right) dy \tag{3.2}$$

This is shown in Fig. 9, for various β and for $a = 0.8$.

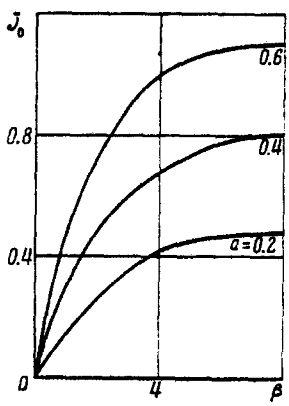

Fig. 8

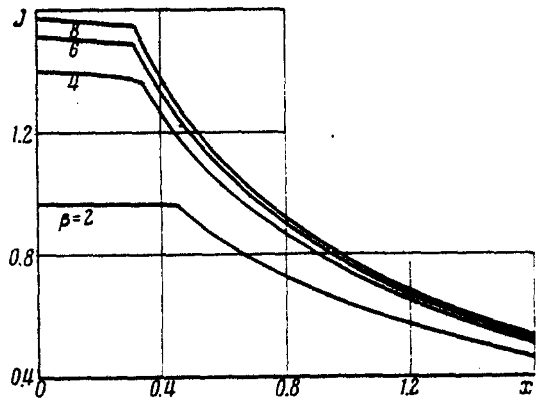

Fig. 9

All curves have initial subregions where the current J necessarily is preserved and basic subregions in which the current dies off to zero. As can be seen from the curves intensive decrease of the current $J(x)$ in the basic subregion takes place for sufficiently large β within a length of channel of the order of the channel height. For small values of β (when the current J_0 is diminished) the relative decrease of J along the length of the channel is less extreme. In the limit $\beta \to 0$ the current J remains constant over the length of the channel. The satisfaction of the condition $J =$ const in the initial subregion serves as a criterion of the accuracy of the calculations performed. It follows from Fig. 9 that the maximum error in the calculation of the integral characteristics does not exceed 3 percent.

It should be pointed out that the flow in the basic subregion of the jet can be investigated with the help of the passage to the limit of $a \to 1$ in the initial problem. In this case it is possible to explain the effects of decrease of current along the length of the channel independently of the flow in the initial subregion. An analysis of this kind was carried through in [5].

In conclusion let us turn our attention to the following situation. As concluded from the results presented the intensity of lateral spreading of the electric jet and the decrease of the current conducted by it along the basic subregion in the channel depend essentially upon the value of the parameter β. For small values β the jet practically does not spread and the influence of the conducting surfaces surrounding the stream (i.e. of the channel walls) remains inappreciable. This shows that when the conditions of supply of a charged component are far from the mode of inductive fluid acquisition, the "control" of the electric jet (the change of its shape, the decrease of current conducted away) is extremely complex. Conversely, when the mode of fluid induction is established, then there exists the possibility of an appreciable change of the electric characteristics of the jet with the aid of guiding surfaces inserted into the flow. Examples of such control by means of the electrical jet were investigated in [6].

The data obtained are applicable to a quantitative prediction of the effects of control by means of the electric jet in each specific case. Let us also point out that instead of the parameter β sometimes it appears more convenient to introduce the value of the nondimensional current imposed in the entrance section.

BIBLIOGRAPHY

1. Bortnikov, Iu. S. and Rubashov, I. B., Some problems of research of the system of equations of electrogasdynamics. Magnitnaia Gidrodinamika, №2, 1968.
2. Vatazhin, A. B., On the deformation of the velocity profile in electrohydrodynamic flow in a plane channel. Izv. Akad. Nauk SSSR, MZhG, №5, 1969.
3. Grabovskii, V. I., Electrohydrodynamic flow in a plane channel with an axial emitter-electrode. PMTF, №6, 1970.
4. Semenova, I. P. and Iakubenko, A. E., The electrohydrodynamic flow of a perfect incompressible fluid in flat and circular channels with zero mobility of charged particles. PMM Vol. 35, №1, 1971.
5. Grabovskii, V. I., Some research problems of electrohydrodynamic jets downstream of a source of charged particles. Izv. Akad. Nauk SSSR, MZhG, №1, 1972.
6. Vatazhin, A. B., Likhter, V. A. and Shul'gin, V. I., Investigation of an electrogasdynamic jet downstream of a source of charged particles. Izv. Akad. Nauk SSSR, M. ZhG, №5, 1971.

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ON THE GENERAL THEORY OF ALMOST SELF-SIMILAR NONSTATIONARY FLOWS

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A system of variational equations is considered, to which nonstationary perfect gas flows differing slightly from the self-similar ones are subject. General expressions are written for the mass, energy and momentum of the material within the perturbed domain and the time-independent summands are extracted therefrom. The first integrals of the variational equations which are extremely simple in form correspond to these summands. The arbitrary constants are selected in such a way that the boundary conditions on the front of a strong shock wave are satisfied automatically.

1. Let us assume that the nonstationary motion of a perfect gas is caused by an explosion or the expansion of a piston. Let the equation giving the position $r_2(t, \varphi, \vartheta)$ of the shock wave propagated over the initially cold gas at rest be represented for large